

Lecture 7: Integration by parts

Cues	Notes
<i>What is it for?</i>	Integrating a <i>product</i> of two functions, when one of them gets simpler if you differentiate it.
<i>The formula, and where it comes from</i>	Integrate the product rule $(uv)' = u'v + uv'$ and rearrange: $\int u v' dx = uv - \int u' v dx.$
<i>How do I choose u?</i>	Take u from whichever comes first in LIATE: L ogs, I nverse trig, A lgebraic, T rig, E xponentials.
<i>Example: $\int x e^x dx$</i>	$u = x$, $v' = e^x$, so $u' = 1$, $v = e^x$: $\int x e^x dx = x e^x - \int e^x dx = e^x(x - 1) + C.$
<i>When do I use it twice?</i>	When the new integral is still a product, e.g. $\int x^2 \sin x dx$: $= -x^2 \cos x + 2x \sin x + 2 \cos x + C.$
<i>Definite integrals</i>	Evaluate the uv term at the limits too: $\int_0^1 x e^x dx = [x e^x]_0^1 - \int_0^1 e^x dx = e - (e - 1) = 1.$
<i>Trick: $\int \ln x dx$</i>	Write it as $\int \ln x \cdot 1 dx$ with $u = \ln x$, $v' = 1$: $\int \ln x dx = x \ln x - x + C.$
<i>Common mistakes</i>	Forgetting $+C$; dropping the minus sign before the second integral; choosing $u = e^x$, which never gets simpler.

Summary

Integration by parts reverses the product rule. Choose u so it simplifies when differentiated (LIATE), apply $\int uv' = uv - \int u'v$, and repeat while the new integral is still a product. Check the answer by differentiating it.